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AN INTRODUCTION TO THE PRICING OF OPTIONS

Jaclyn Grimshaw wrote this note under the supervision of Professors Walid Busaba and Zeigham Khokher solely to provide material for class discussion. The authors do not intend to illustrate either effective or ineffective handling of a managerial situation. The authors may have disguised certain names and other identifying information to protect confidentiality.

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A derivative is a financial instrument that derives its value in part from the characteristics of the underlying security on which it is based. In valuing the premium to be paid to purchase a derivative, the underlying security itself is, however, just one of several determinants. This note examines the various factors that contribute to an option's value and the relative impact of each of these factors. Two of the more commonly used option pricing models are also introduced: the Black-Scholes model (see Exhibit 1) and the binomial option pricing model (see Exhibit 2).

OPTION PREMIUM DETERMINANTS

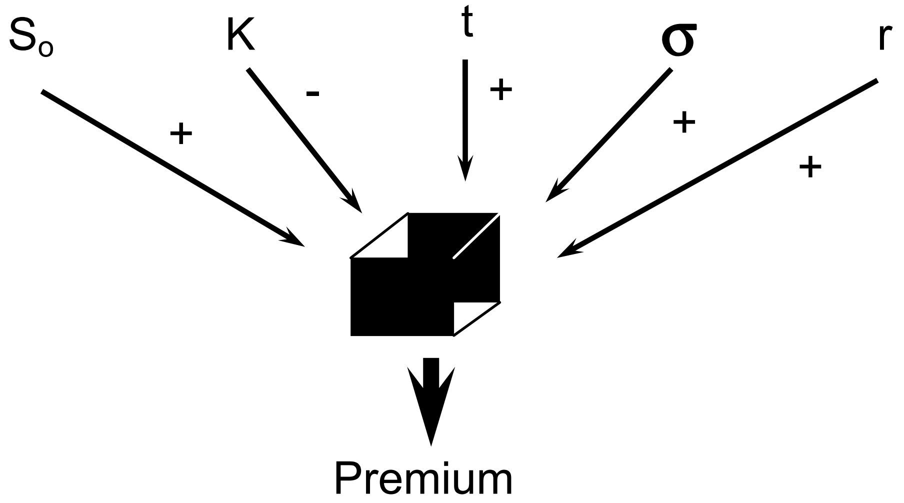
The *intrinsic value* of an option is the amount by which an option is *in-the-money*, or, in the case of a call option, the amount by which the current market price exceeds the exercise price. One might mistakenly assume then that the premium to be paid to purchase an option should equal only its intrinsic value. If this supposition were true however, then an option that was out-of-the-money would be worthless; in reality, options are often issued or written, and subsequently traded, with exercise prices that are *at-* or *out-of-the-money*. The intrinsic value is thus just one component of an option's value; the other component is referred to as its *time value* — where time value equals the option premium less its intrinsic value. The time value of an option premium is a function of the time remaining until expiration together with any restrictions on when exercise may occur, the historical trading volatility of the underlying security and the effective cost of holding a position in the underlying security, including both interest rates foregone and dividends to be paid.

Table 1 summarizes the relative effect of each of the various determinants on the value of a European call or put option. For a call option, as the price of the underlying security increases, the intrinsic value also increases and, thus, overall the value or premium to be paid for the option increases.

Table 1 — Factors Influencing an Option's Premium

	Effect on:	
Determinant	**Value of Call**	**Value of Put**
Current stock price, S_o	Increase	Decrease
Exercise price, K	Decrease	Increase
Time to expiry, t	Increase	Increase
Share volatility, σ	Increase	Increase
Interest rate, r	Increase	Decrease
Dividends paid	Decrease	Increase

Figure 1 — Option Premium Pricing Determinants



Intrinsic Value

The positive differential between the price of the underlying security and the option's exercise price comprise its intrinsic value. The intrinsic value must be non-negative and when the option is out-of-the-money, the intrinsic value is simply zero.

$$\textit{Intrinsic Value of Call} = \max(S_o - K, 0)$$
$$\textit{Intrinsic Value of Put} = \max(K - S_o, 0)$$

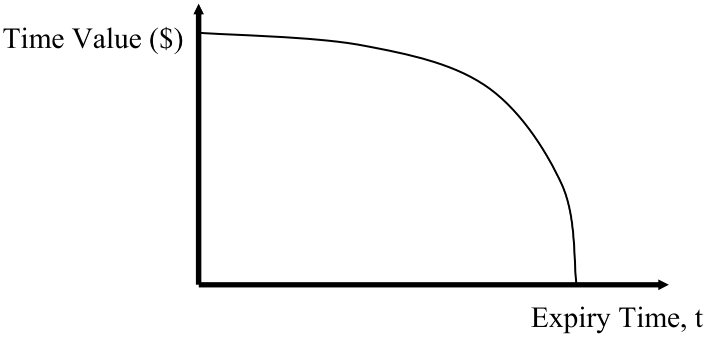
If at expiration, when the time value component equals zero, the underlying security was trading at $25 and an investor held a call option on that security with an exercise price of $15, then the payoff of the call would be $10. If the exercise price, which was set when the option was initially written or issued, were instead $30, then the payoff of the call option would be zero as the holder of the out-of-the-money option would allow the option to expire without exercising it.

Note that profit to the option holder does not equal the payoff because a premium was paid to enter into the option contract initially. Instead, profit is equal to payoff less the premium paid to purchase the option. If an option is out-of-the-money at expiration, the option buyer incurs a net loss equal to the premium paid.

Time Value

The probability of an option being in-the-money at expiration increases with a longer time to maturity. In theory, an investor must then be willing to pay a higher valuation for the increased probability of positive payoff; thus, an option's time value increases with time remaining to maturity.[1] The time value decay diagram in Figure 2 illustrates that an option's time value decreases at an accelerating rate over its life, concluding with a value of zero at expiration.

Figure 2 — Time Value Decay



Volatility

For the purpose of valuing an option premium, volatility refers to the degree of change, in value of the underlying security over time. If a security has historically had high volatility, it can be argued that it is more likely to also experience greater price movements in the future; thus, an option written on the security is more likely to be in-the-money at the time it is exercised. So options on underlying securities with higher historical volatility command higher premiums than options written on securities with lower historical volatility.

Interest Rates

Interest rates play a role in an option premium because of the concept of the opportunity cost of cash. Purchasing a call option effectively delays the outlay of cash associated with purchasing the underlying security itself. Call premiums thus increase when prevailing market interest rates are higher due to the increased opportunity cost associated with the ability to invest money that is not otherwise tied up in the underlying security. Put premiums react in the opposite manner and tend to decrease with increasing interest rates. A covered put buyer holds the underlying security on which the put is written and thus has a lost opportunity to invest in an interest bearing instrument.

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[1] *Ceteris paribus, i.e., if all other inputs are held constant.*

Dividends

When dividends are paid to shareholders of an underlying security, the price of the security typically decreases by the amount equivalent to the dividend paid per share. This price decrease also results in a corresponding decrease in value for the call option relative to the size of the anticipated dividends and an increase in the value of the put positively related to the size of the anticipated dividends.

OPTION EQUIVALENTS

The discounted cash flow (DCF) model is a common approach to valuing many investments. In its simplest form, a DCF is the sum of cash flows over the period of the investment, adjusted for the time value of money. The problem with applying a DCF valuation to determine an option premium is that the appropriate discount rate is constantly changing as a result of the option's dynamic risk profile.[2]

To more accurately calculate the true value of an option, two different pricing models are commonly used: the Black-Scholes model and the binomial model. Both models value an option premium by setting up what is referred to as an *option equivalent* investment consisting of common stock and a bank loan. This combination of common stock and borrowed funds is also known as a *replicating* or *tracking portfolio*. The payoff of the replicating portfolio is constructed in such a way that it is identical to the payoff of the option investment; hence, the option premium must then be equal to the value of the replicating portfolio.

Consider again the three-month European call option with the \$25 exercise price. Assume that the security's current price is also \$25, and there is an equal probability that in three months' time the price can either increase 25 per cent to \$31.25 or decrease 20 per cent to \$20.00. The resulting payoff of the call option at maturity is either \$6.25 or zero — the option's intrinsic value, $\max(S_o - K,0)$.

In creating the option equivalent, the number of shares required to replicate one call option is known as the hedge ratio or *option delta,* Δ:[3]

$$\begin{aligned}\Delta &= \frac{spread\ of\ option\ prices}{spread\ of\ share\ prices}\\ &= \frac{(\$6.25-\$0)}{(\$31.25-\$20.00)}\\ &= 0.5556\end{aligned}$$

Table 2 shows that the replicating portfolio of purchasing Δ shares and borrowing the funds to finance the purchase has the same payoff as purchasing the \$25 call option.

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[2] *Note that an option is always riskier than the underlying security; the degree of risk depends on the difference between the option's exercise price and the underlying security's market price and the time remaining to expiry.*

[3] *A put option delta is equal to the delta of a call option minus 1.*

Table 2 — Call Option Equivalent Replicating Portfolio

Security Price at Maturity	**\$20.00**	**\$31.25**
Δ Shares Purchased (0.5556)	\$11.11	\$17.36
Loan plus Interest[4]	–\$11.11	–\$11.11
Total Payoff	**\$0.00**	**\$6.25**

Because the two investments have identical terminal payoffs, they must also have the same value. By determining the value of the option equivalent, the value of the call can be calculated as follows:

$$\begin{aligned} &Value\ of\ \ Call = Value\ of\ \ Option\ Equivalent \\ &= [\Delta \times S_o] - PV(bank\ loan) \\ &= 0.5556\ \times\ \$25.00 - \frac{\$11.11}{(1+1\%)} \\ &= \$2.89 \end{aligned}$$

BLACK-SCHOLES OPTION PRICING MODEL

The option equivalent discussed in the previous section priced an option by setting up an investment portfolio that replicated the payoff of an option. The drawback of the replicating portfolio is that it simplifies the underlying security's price to two discrete movements during its life: one up and one down.

Pricing models have been developed to better replicate the payoff of an option by incorporating an increasing number of possible price movements over the life of the option. The binomial model arrives at an option's price using a discrete model based on a finite number of outcomes for a finite number of periods (see Exhibit 2). The Black-Scholes model is a continuous time alternative to the binomial model, which accounts for an infinite number of stock price outcomes.

Similar to the replicating portfolio, the Black-Scholes model also calculates the value of a call as:

$$[\Delta \times S_o] - PV(bank\ loan)$$

For the Black-Scholes formula and an explanation of how the delta and the bank loan values are calculated, see Exhibit 1. Most investors today use online Black-Scholes options models or financial calculators to compute option prices.

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[4] *The prevailing risk-free rate is assumed to be 4 per cent per annum or approximately 1 per cent per quarter. The amount borrowed from the bank is equal to the present value of the difference between the payoff from the option and the payoff of the delta shares. Using our example, PV(\$25.00 – 0.5556 × \$25.00) = \$11.11 / 1.01 = \$11.00. Interest is reapplied at the same rate, resulting in the loan plus interest equal to \$11.11.*

The inputs for the Black-Scholes model are the same determinants as introduced above:

S_o = current price of underlying stock
K = option exercise price
t = time to maturity (in years)
σ = volatility, which can be calculated from standard deviation of returns (annualized)
r = risk-free interest rate

One key advantage of the Black-Scholes formula is the speed with which it can calculate the value of an option. However, because it is only capable of calculating the option premium at one point in time, at expiration, the Black-Scholes formula is less than ideal for valuing American options. The binomial model discussed in Exhibit 2 can be used to more accurately price American options because this model allows the investor to check value at various points of the option's life for the possibility of optimal early exercise. Further, the generic Black-Scholes formula discussed in this note assumes that the option is written on a non-dividend-paying security. Subsequent models have been developed that account for dividends by incorporating expected dividend yields.

Several other key assumptions are embedded in the generic Black-Scholes formula. The formula was designed for valuing options on securities with market liquidity and constant variance. It also assumes a constant risk-free interest rate. For volatility, prudent investors use the average expected volatility. For the risk-free rate, investors can use the yield of a zero coupon bond set to mature on the same date as the option is to expire.

Exhibit 1

THE BLACK-SCHOLES FORMULA

The Black-Scholes formula calculates the value of a call option as:

$$Value\ of\ Call = [N(d_1) \times S_o] - [N(d_2) \times PV(K)]$$

where, d_1 = $$\frac{\ln\left(S_o/K\right) + \left(r + \sigma^2/2\right) \cdot t}{\sigma\sqrt{t}}$$

d_2 = $d_1 - \sigma\sqrt{t}$

N( ) = cumulative normal probability function

S_o = current price of underlying stock

K = option exercise price

PV(K) = K $e^{-r \cdot t}$; the present value of the exercise price discounted at the risk-free rate, using continuous compounding[1]

t = time to maturity (in years)

σ = volatility, which can be calculated from standard deviation of returns (annualized)

r = risk-free interest rate

In the Black-Scholes formula, the delta of the call is equal to $N(d_1)$, and the “bank loan” is the second term, $N(d_2) \times PV(K)$.

The cumulative normal probability function (N(d)) states the probability of a normally distributed random variable being equal or less than d. Once d_1 and d_2 have been calculated, a normal distribution table or the NORMSDIST function in Excel can be used to find the probability functions required for the Black-Scholes formula.

A Numerical Example — Call Option

Although the derivation of the Black-Scholes formula is complex and beyond the scope of this discussion, the formula itself is quite easy to apply given that all of the required inputs are readily available. For instance, consider a call with an exercise price of $70.00 and current stock price of $68.80. It will be assumed that the standard deviation is 40 per cent, time to maturity is a fiscal quarter or 0.25 years, and the current prevailing risk-free rate per annum is 5 per cent.

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[1] ***e** is the inverse of **ln**, the natural logarithm, and is equal to 2.7183. For computational purposes, **e** can be found on most scientific calculators and is the function EXP() within Excel.*

Exhibit 1 (continued)

The first step in using the Black-Scholes formula is to calculate d_1 and d_2:

$$d_1 = \frac{\ln\left(S_o/K\right) + \left(r + \sigma^2/2\right)\cdot t}{\sigma\sqrt{t}}$$

$$= \frac{\ln\left(68.80/70.00\right) + \left(5\% + 0.4^2/2\right)\cdot \frac{3}{12}}{0.4\sqrt{\frac{3}{12}}}$$

$$= 0.07604$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$= 0.07604 - 0.4\sqrt{\frac{3}{12}}$$

$$= -0.12396$$

The second step is to find $N(d_1)$ and $N(d_2)$:

$$N(d_1) = 0.5303$$

$$N(d_2) = 0.4507$$

Thus,

$$Value\ of\ Call = [N(d_1) \times S_o] - [N(d_2) \times PV(K)]$$

$$= [0.5303 \times \$68.80] - [0.4507 \times \$70.00e^{-0.05\cdot\frac{3}{12}}]$$

$$= \$5.33$$

A Numerical Example — Put Option

If the option to be valued is a put rather than a call, the value of the put premium can be determined by simply applying the Put-Call Parity formula to an equivalent call option:

Value of Call + PV(Exercise Price) = Value of Put + Underlying Stock Price

Value of Put = Value of Call + PV(Exercise Price) – Underlying Stock Price

$$= \$5.33 + \$70.00e^{-0.05\cdot\frac{3}{12}} - \$68.80$$

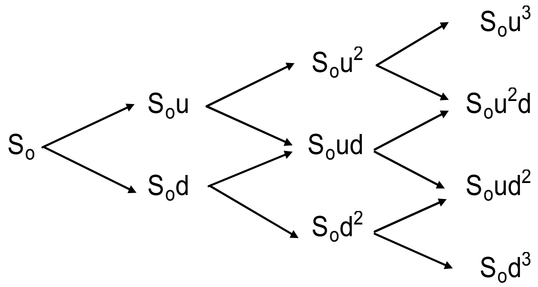
$$= \$5.66$$

Exhibit 2

THE BINOMIAL OPTION PRICING MODEL

This note’s example of the option equivalent replicating portfolio assumed only two possible price movements over the three-month period to expiry: one up and one down. In reality, the security’s price moves continuously; simulating a greater number of price changes over smaller intervals is a better approximation. Binomial trees can be used to represent the possible paths that the price can take over the life of the option. For example, if instead of one price movement in the three-month period, there were three periods of equal price movements (i.e., monthly movements), the binomial tree below would result.

Price Movement of Underlying Stock in a Three-Step Binomial Tree



The binomial tree method uses a decision tree to work backwards from some future point in time (most often the expiry date) to determine the present value of an option, f (see the figure below). For *n* intervals, the first step is to calculate the option values at interval *n* as the intrinsic value. Then, using these calculated option values, the option values at the subsequent interval, n–1, are quantifiable. The process of moving backwards through the decision tree is repeated until the single option value for the current period is determined.

Option Prices in a Three-Step Binomial Tree

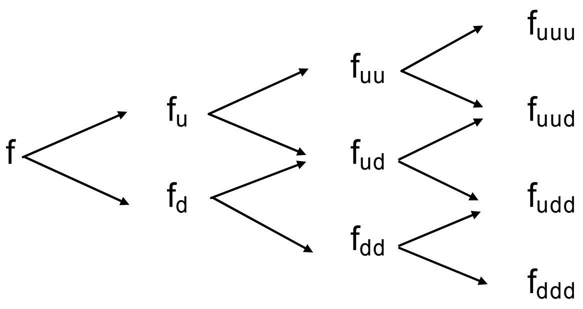


Exhibit 2 (continued)

In addition to knowing the number of intervals n over which to calculate the option's value, the volatility σ of the underlying stock must also be known. Together these two pieces of data are used to arrive at the price movement per interval, u and d.

(upside change + 1) = $u = e^{\sigma\sqrt{h}}$
(downside change + 1) = $d = 1/u$

where

σ = standard deviation of continuously compounded stock returns (annualized)
h = period interval, as a fraction of a year

Further, the probability of up and down movements must be calculated:

up movement probability = $p = (a-d)/(u-d)$
down movement probability = $1 - p$

where

a = e^{rh} = growth factor
r = risk-free interest rate

Using the calculated upside and downside changes and probabilities of said changes occurring, the option value become:

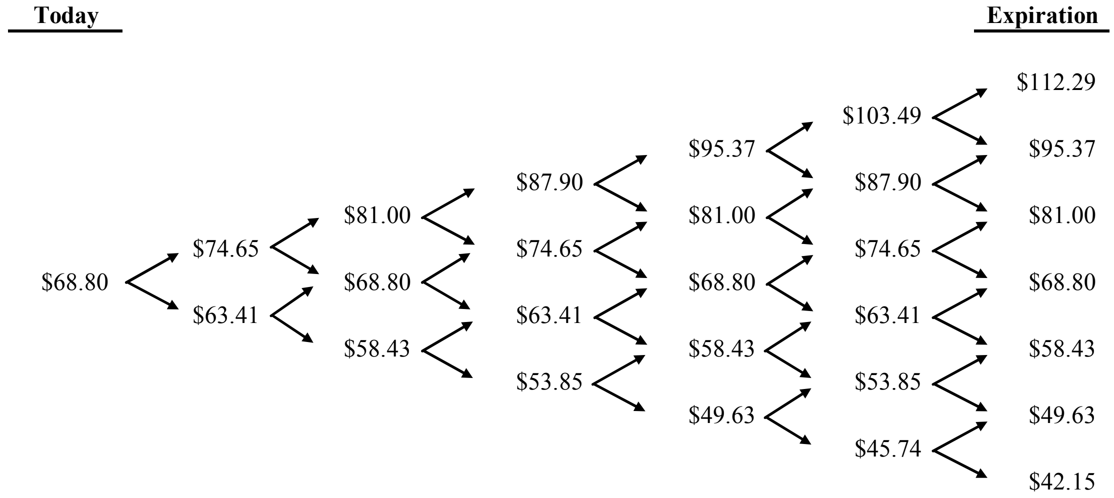
$$f = e^{-rh}\left[pf_u + (1-p)f_d\right]$$

A Numerical Example — Call Option

To calculate the value of a three-month call option on a non-dividend-paying stock, the first step is to determine the projected stock prices at each semi-monthly interval (see the figure below). Note that for this example the pricing determinants from the previous example are used over six equal intervals. At the assumed standard deviation of 40 per cent, the resulting and downside changes per interval (or 1/24th of a year) are u=1.085 and d=0.922, respectively.

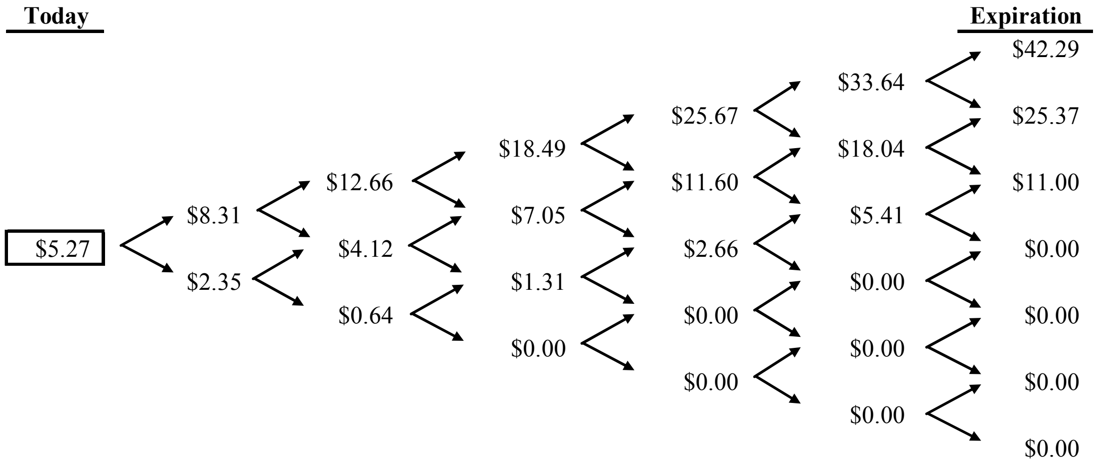
Exhibit 2 (continued)

Step 1. Binomial Tree of Stock Price Movements



The second step in solving the binomial tree is to determine the call option price, *f*. Starting at the final node of the binomial tree, the expiration date, the option's price consists solely of its intrinsic value (see Step 2 below). Recall that for a call option the intrinsic value is calculated as $\max(S_o - K, 0)$.

Step 2. Binomial Tree of a Three-month Call Option on a Non-Dividend Paying Stock



Subsequent intervals are calculated as the present value of the expected option price from the previous interval, working from expiration to the present. The probabilities must be applied to determine the correct expected values (in this example, p=0.492 and 1–p=0.508). As an example, for the uppermost node immediately before expiration the option price is calculated as:

$$\frac{(\$42.29 \times 0.492 + \$25.37 \times 0.508)}{(1+5\%)^{1/24}}$$
$$= \$33.64$$

The remaining intervals are calculated in the same manner, and the value of the call option calculated using the six interval binomial model is \$5.27. Comparing this value to the value calculated using the Black-Scholes model, reveals a 1 per cent difference in the option premiums calculated. For European options on non-dividend paying securities, the binomial model converges on the Black-Scholes model as the number of intervals used increases.

Put Option

The binomial model procedure followed above can also be easily applied to price a put option premium. The only difference in determining a put premium with this model lies in the first option value, *f*, where a put premium's intrinsic value is max(K – S_o, 0). The put premium calculated with the binomial model is \$5.60 for this example.